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SCATTER FACTOR AND RELIABILITY OF AIRCRAFT STRUCTURES

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16. Abstract

The concept of time to first failure is utilized to perform a parameter study of scatter factors of aircraft structures. The Weibull distribution is used for estimation of characteristic and certifiable lives. Scatter factors for various Weibull-shaped parameters, fleet sizes and level of reliabilities are calculated. It is concluded that the currently used range of scatter factors (2 through 4) is too narrow for the estimation of a "safe" life; also that a safe and economical design for structural materials with shape parameters less than 2 does not seem feasible except for very small fleet sizes and low levels of reliability.

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NOMENCLATURE

α	=	Weibull distribution shape or scatter controlling parameter
β	=	Weibull distribution scale parameter or characteristic life
β_1	=	Weibull distribution scale parameter of time to first failure
F_{Y_1}	=	Cumulative distribution of time to first failure
F_Y	=	Cumulative distribution of population
n	=	test sample size
n_f	=	number of failure observations in the test sample
N	=	fleet size
R	=	fleet reliability
S_R	=	scatter factor for a specified reliability level
Y_i	=	time to failure variable
Y_1	=	time to first failure
Y_R	=	safe life for a specified reliability

1. INTRODUCTION

Conventional procedures of reliability estimation are based on the assumption of arbitrary scatter factors of between 2 and 4, which applied to an "estimated" life determined from development tests, are supposed to produce the safe and certifiable life. The probability of occurrence of individual lives shorter than the design life in the fleet of airplanes to which this estimate refers remains thus completely undefined but is presumably zero. No statements concerning the reliability level of the design can be made, since no confidence level can be attached to the selected scatter factor, nor can any statistical significance be attached to the "estimated" life.

A basic improvement of the conventional procedures based on arbitrary scatter factors applied to an ill-defined "expected life" was achieved through the application of the concept of the "expected time to first failure in a fleet" of structures subject to progressive fatigue damage, as shown by Freudenthal[1*]. This research formed the basis of a further investigation by Whittaker and Besuner[2] on a reliability analysis plan for application to aluminum alloy structural fatigue performance which was compared to the current fixed-scatter-factor procedure for determining the safe life of a structural detail.

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*References at the end of report in numerical order.

The present study attempts to show the interrelation between the various parameters, such as fleet size, shape-parameter, level of reliability, in their influence on the scatter factor.

The third asymtotic extremal distribution (Weibull) of smallest values was used for estimation of the certifiable life.

2. MATHEMATICAL FORMULATION

2.1 Estimation of Characteristic Life

With the shape-parameter assumed to be known, one depends on the full scale fatigue test to obtain the most likely value of the parent population's central tendency or scale parameter. It is to be recognized that the occurrence of high-time outliers in this testing could lead to overly high safe-life estimator. It can be argued that these high-time outliers are not part of the parent population and can therefore be censored. Fortunately, as has been observed in the data, large, complex, multicomponent specimens are not characterized by outliers to anywhere near the extent that simple monolithic specimens are.

As shown by Mann[3], the Maximum Likelihood Estimator (MLE) provides a best estimate for the two parameter Weibull distribution.

The point estimate of the Weibull characteristic life[2,3] β, with shape-parameter α known, is

$$\hat{\beta} = \left[\frac{1}{n_f}\left(\sum_{i=1}^{n_f} Y_i^{\alpha} + (n - n_f)\, Y_{(n_f)}^{\alpha}\right)\right]^{1/\alpha} \qquad (1)$$

which considers the case of single-stage censoring at the n_f^{th} failure time only. Equation (1) gives an unbiased uniformily minimum variance estimator for which the sampling distribution is known. For uncensored or complete sample, equation (1) reduces obviously to

$$\hat{\beta} = \left[\frac{1}{n_f}\left(\sum_{i=1}^{n_f} Y_i^{\alpha}\right)\right]^{1/\alpha} \tag{2}$$

For this purpose it is also possible, if desired, to calculate the lower bound interval estimate. The sampling distribution of the Weibull $\hat{\beta}$ estimator is then given in terms of a probability statement of confidence level, accounting for the finite sample size.

2.2 Estimation of Certifiable Life

If avoidance of a single catastrophic failure is a design requirement, the time to first failure will be the significant design criterion.

From basic order statistical considerations[1]

$$1 - F_{Y_1}(y_1) = [1 - F_Y(y)]^N \tag{3}$$

and

$$1 - F_{Y_1}(y_1) = \exp\left[-N\left(\frac{y_1}{\beta}\right)^{\alpha}\right]$$

$$1 - F_{Y_1}(y_1) = \exp\left[-\frac{y_1}{\beta_1}\right]^{\alpha} \tag{4}$$

where $\beta_1 = \beta N^{-\frac{1}{\alpha}}$, which shows that for this distribution the function $(1 - F_{Y_1}(y_1))$ is of the same extremal type as $(1 - F_Y(y))$, with identical shape-parameter α but characteristic value reduced by $(N^{-\frac{1}{\alpha}})$. Equation (4) can be solved for y_1,

$$y_1 = \beta_1 \{\ell n(\frac{1}{1- F_{Y_1}(y_1)})\}^{1/\alpha} \tag{5}$$

Setting $1- F_{Y_1}(y_1) = R$, the equation above can be rewritten

$$Y_R = \beta_1 \{\ell n(\frac{1}{R})\}^{1/\alpha} \tag{6}$$

where Y_R is the safe life for a specified fleet reliability. Equation (6) can also be called the "inverse reliability function."

2.3 The Scatter Factor

The scatter factor accounting for both the fleet exposure and specified fleet reliability R is simply

$$S_R = \frac{\hat{\beta}}{Y_R} \tag{7}$$

as shown in figure 1.

3. COMPUTATIONAL PROCEDURE

In order to perform a parameter study a general computer program was developed and listed in Appendix A. The necessary input information is summarized in Table 1.

The program can handle both censored and uncensored samples resulting from fatigue tests for the evaluation of the MLE of $\hat{\beta}$.

Various shape parameters α can be specified according to the types of materials being used. Furthermore various fleet sizes can also be specified. As final input parameter the various levels of desired reliability can be specified.

4. RESULTS OF DATA ANALYSIS

As an example, item 56 of reference 2 was chosen at random which is an uncensored sample of size 3. The input information is shown in table 1. The values for the shape parameters α vary from 1.0 up to 15.0, the fleet sizes from 5 up to 5000. Levels of reliability range from 0.50 to 0.99. Tables 2 through 13 summarize the results in terms of scatter factors.

5. DISCUSSION OF RESULTS

The point estimate of the central tendency or characteristic life of the two-parameter Weibull distribution shows an increasing confidence in the higher life test results with increasing α (figure 2). This tendency is quite obvious since a large value of α for the Weibull distribution corresponds to a small scatter in data since $\sigma(\ell nY) = \pi/\alpha\sqrt{6}$.

The study also reveals the influence of fleet size, level of reliability as well as the Weibull shape parameter on the scatter factor.

Figures 3 to 7 show the relationship of the reliability R to the scatter factor S_R for the fleet sizes N = 5, 50, 250, 1000 and 5000 respectively. Each plot shows a family of curves,

each curve corresponds to a certain α. One can note a considerable faster increase of scatter factors in the reliability interval of 0.95 and 0.99 than in the lower intervals.

The results furthermore show a rapid increase in the values for scatter factors with increasing fleet size and level of reliability. This can be illustrated by a simple example.

For a metal with a predetermined α of 2 and a desired reliability level of 0.75 the scatter factor S_R for a fleet of $N = 5$ will be 4.27; for a fleet of $N = 250$, S_R will already be 29.50 and for $N = 5000$, $S_R = 131.83$. If a level of reliability of 0.99 is desired, the values for S_R increase to 22.3, 157.72 and 705.33 respectively.

The scatter factors for metals with shape factors $\alpha < 2$ at higher levels of reliability (> 0.90) reach extremely high values especially for larger fleet sizes. See tables 2 to 5 and figures 3 to 7.

Figures 8, 9 and 10 show a logarithmically linear relationship between fleet sizes N and scatter factors S_R for various levels of reliability (0.90, 0.75 and 0.50). For example: on the basis of a reliability of 0.90 a metal with $\alpha = 4$ will yield higher values for scatter factors than a metal of $\alpha = 3$ on the basis of a reliability of 0.5. But this will be the case only up to a fleet size of $N = 200$; if $N \geq 200$ the reverse is true.

Finally in figure 11 the relationship of the Weibull shape parameter and the scatter factor is demonstrated. The figure shows various families of curves. Their constant parameters are the fleet size N and the reliability R. It is interesting to note that for a given shape parameter α for example a fleet-size of N = 250 with a level of reliability of 0.50 results in lower scatter factors than a fleet size of N = 50 with a level of reliability of 0.90. The same relationship is true for both fleet sizes N = 250 and 1000, also for N = 1000 and 5000.

Figure 11 also implies that the conventionally used scatter factors $S_R \leq 4$ associated with an acceptable reliability level R = 0.9 would require a shape parameter $\alpha \geq 4.5$ even for a fleet size N = 50 and $\alpha \geq 6.0$ for a fleet size N = 250 that is more representative of real conditions. Shape parameters of this magnitude imply a level of material-, production-, and mission-control that does not reflect realistic conditions of aircraft manufacture and utilization. Nor does a reliability level R = 0.90 reflect the implied expectation of the user. Thus, under the assumption[2] that $\alpha = 4$ is a representative value for aluminum structures and that a reliability level R = 0.95 better reflects the expectation of the user, a scatter factor $S_R = 8.4$ (rather than $S_R = 4.0$) would define the "certifiable" life for realistic fleet size of N = 250. This value of the scatter factor should therefore be used for long range transport planes of current design for which $\alpha = 4$ is a representative value.

For advanced designs using titanium or high-strength steel alloys preliminary observations suggest that representative values of α are below $\alpha = 4$, so that larger values of S_R for the specification of a certifiable life must be expected.

6. CONCLUSIONS

(a) The study shows clearly that a safe and economical design for metals with shape parameters $\alpha < 2$ does not seem to be feasible if the design is based on the realistic concept of time to first failure, except for very small fleet sizes and low levels of reliability.

(b) Scatter factors for establishment of a safe life used in procurement of fatigue sensitive aircraft are too narrow even for current designs.

REFERENCES

(1) FREUDENTHAL, A. M., "Reliability Analysis Based on Time to the First Failure," Conference Paper Presented at the Fifth I.C.A.F. Symposium, Melbourne, Australia, May 22-24, 1967.

(2) WHITTAKER, I.C. and BESUNER, P.M., "A Reliability Analysis Approach to Fatigue Life Variability of Aircraft Structures." AFML-TR-69-65, Air Force Materials Laboratory, Aeronautical Systems Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio.

(3) MANN, N. R., "Point and Interval Estimation Procedures for the Two-Parameter Weibull and Extreme-Value Distributions," Technometrics 10, May 1968, pp. 231-76.

INPUT

SAMPLESIZE= 3 NUMBER FAILED= 3 ALFAS= 12 GAMAS= 11 NUMBER OF FLEETS= 10

SAMPLES

4000.0 5400.0 5500.0

FAILURES

4000.0 5400.0 5500.0

ALFAS

1.00	1.20	1.60	1.80	2.00	3.00	4.00	5.00
6.00	8.00	10.00	15.00				

FLEETSIZES

5 10 50 100 200 250 500 1000 2000 5000

	GAMA	W
1	0.010	-1.530
2	0.050	-1.090
3	0.100	-0.800
4	0.150	-0.660
5	0.200	-0.450
6	0.250	-0.290
7	0.300	-0.190
8	0.350	-0.050
9	0.400	0.100
10	0.450	0.210
11	0.500	0.390

Table 1: Summary of Input Data

ALFA= 1.00

BETA= 4966.7

GAMA	N= 5	N= 10	N= 50	N= 100	N= 200	N= 250	N= 500	N=1000	N=2000	N=5000
0.010	497.496	994.993	4974.961	9949.926	19899.860	24874.820	49749.650	99499.250	198998.600	497496.500
0.050	97.479	194.958	974.787	1949.575	3899.151	4873.938	9747.871	19495.740	38991.500	97475.680
0.100	47.456	94.912	474.561	949.122	1898.245	2372.806	4745.609	9491.219	18982.440	47456.120
0.150	30.766	61.531	307.657	615.313	1230.627	1538.283	3076.568	6153.129	12306.260	30765.660
0.200	22.407	44.814	224.071	448.142	896.284	1120.355	2240.710	4481.422	8962.840	22407.100
0.250	17.380	34.761	173.803	347.606	695.212	869.015	1738.030	3476.061	6952.117	17380.300
0.300	14.018	28.037	140.184	280.367	560.735	700.918	1401.837	2803.674	5607.348	14018.360
0.350	11.607	23.214	116.068	232.135	464.271	580.339	1160.677	2321.355	4642.711	11606.770
0.400	9.788	19.576	97.881	195.762	391.523	489.404	978.808	1957.615	3915.232	9788.078
0.450	8.363	16.727	83.635	167.270	334.539	418.174	836.349	1672.697	3345.396	8363.484
0.500	7.213	14.427	72.135	144.270	288.539	360.674	721.348	1442.695	2885.392	7213.477

Table 2: Scatter Factors for $\alpha = 1.0$

ALFA= 1.20

BETA= 4976.4

GAMA	N= 5	N= 10	N= 50	N= 100	N= 200	N= 250	N= 500	N=1000	N=2000	N=5000
0.010	176.736	314.908	1204.089	2145.444	3822.747	4603.984	8203.367	14616.730	26044.060	55888.910
0.050	45.439	80.962	309.570	551.591	982.823	1183.678	2109.076	3757.943	6695.895	14368.960
0.100	24.941	44.439	169.920	302.762	539.461	649.709	1157.650	2062.697	3675.311	7886.977
0.150	17.380	30.968	116.410	210.983	375.929	452.756	806.719	1437.409	2561.174	5496.113
0.200	13.345	23.778	90.919	161.999	288.650	347.640	619.425	1103.689	1966.551	4220.090
0.250	10.799	19.242	73.572	131.091	233.578	281.313	501.243	893.112	1591.346	3414.929
0.300	9.028	16.086	61.506	109.591	195.268	235.174	419.033	746.631	1330.347	2854.840
0.350	7.714	13.744	52.552	93.638	166.844	200.941	358.036	637.947	1136.693	2439.272
0.400	6.692	11.924	45.595	81.241	144.754	174.337	310.634	553.485	986.199	2116.321
0.450	5.870	10.460	39.993	71.260	126.972	152.920	272.473	485.491	865.047	1856.335
0.500	5.189	9.247	35.355	62.996	112.246	135.185	240.873	429.196	764.723	1641.047

Table 3: Scatter Factors for $\alpha = 1.2$

ALFA= 1.60

BETA= 4995.3

GAMA	N= 5	N= 10	N= 50	N= 100	N= 200	N= 250	N= 500	N=1000	N=2000	N=5000
0.010	48.472	74.754	204.406	315.237	486.161	558.919	861.971	1329.342	2050.126	3634.905
0.050	17.501	26.991	73.802	113.818	175.532	201.802	311.220	479.967	740.211	1312.406
0.100	11.160	17.212	47.063	72.582	111.936	128.688	198.464	306.074	472.030	836.916
0.150	8.512	13.128	35.896	55.359	85.375	98.152	151.370	233.445	360.021	638.323
0.200	6.982	10.768	29.444	45.408	70.029	80.509	124.163	191.485	295.310	523.589
0.250	5.957	9.187	25.121	38.742	59.748	68.690	105.934	163.372	251.955	446.720
0.300	5.208	8.032	21.963	33.871	52.236	60.054	92.616	142.833	220.279	390.558
0.350	4.629	7.138	19.518	30.102	46.423	53.370	82.308	126.937	195.764	347.092
0.400	4.161	6.417	17.546	27.060	41.732	47.978	73.992	114.111	175.984	312.022
0.450	3.771	5.816	15.903	24.526	37.825	43.486	67.064	103.427	159.507	282.808
0.500	3.438	5.303	14.499	22.361	34.485	39.646	61.142	94.294	145.421	257.834

Table 4: Scatter Factors for α = 1.6

ALFA= 1.80

BETA= 5004.6

GAMA	N= 5	N= 10	N= 50	N= 100	N= 200	N= 250	N= 500	N=1000	N=2000	N=5000
0.010	31.493	46.287	113.181	166.345	244.484	276.750	406.750	597.814	878.628	1461.783
0.050	12.734	18.715	45.762	67.258	98.851	111.898	164.460	241.713	355.253	591.039
0.100	8.536	12.546	30.378	45.089	66.268	75.014	110.251	162.040	238.150	396.223
0.150	6.710	9.861	24.113	35.440	52.088	58.962	86.659	127.366	187.194	311.437
0.200	5.626	8.269	20.219	29.717	43.676	49.441	72.665	106.798	156.965	261.145
0.250	4.886	7.181	17.558	25.806	37.927	42.933	63.100	92.741	136.304	226.771
0.300	4.336	6.372	15.582	22.901	33.658	38.100	55.997	82.301	120.960	201.243
0.350	3.904	5.738	14.030	20.621	30.307	34.307	50.422	74.106	108.917	181.206
0.400	3.551	5.219	12.763	18.758	27.569	31.208	45.867	67.412	99.076	164.837
0.450	3.254	4.783	11.695	17.188	25.262	28.596	42.029	61.771	90.787	151.044
0.500	2.997	4.405	10.772	15.832	23.269	26.340	38.713	56.898	83.625	139.128

Table 5: Scatter Factors for $\alpha = 1.8$

ALFA= 2.00

BETA= 3013.0

GAMA	N= 5	N= 10	N= 50	N= 100	N= 200	N= 250	N= 500	N=1000	N=2000	N=5000
0.010	22.305	31.544	70.533	99.749	141.067	157.718	223.046	315.435	446.093	705.334
0.050	9.873	13.963	31.222	44.154	62.443	69.814	98.731	139.627	197.463	312.216
0.100	6.889	9.742	21.784	30.808	43.569	48.711	68.888	97.423	137.777	217.844
0.150	5.547	7.844	17.540	24.806	35.080	39.221	55.467	78.442	110.934	175.402
0.200	4.734	6.694	14.969	21.169	29.938	33.472	47.336	66.943	94.672	149.690
0.250	4.169	5.896	13.183	18.644	26.367	29.479	41.690	58.958	83.379	131.834
0.300	3.744	5.295	11.840	16.744	23.680	26.475	37.441	52.950	74.882	118.399
0.350	3.407	4.818	10.773	15.236	21.547	24.090	34.069	48.180	68.137	107.735
0.400	3.129	4.424	9.893	13.991	19.787	22.122	31.286	44.245	62.572	98.935
0.450	2.892	4.090	9.145	12.933	18.290	20.449	28.920	40.899	57.839	91.452
0.500	2.686	3.798	8.493	12.011	16.986	18.991	26.858	37.983	53.716	84.932

Table 6: Scatter Factors for α = 2.0

ALFA= 3.00

BETA= 5056.4

GAMA	N= 5	N= 10	N= 50	N= 100	N= 200	N= 250	N= 500	N=1000	N=2000	N=5000
0.010	7.924	9.983	17.071	21.508	27.099	29.191	36.779	46.338	58.383	79.237
0.050	4.602	5.798	9.915	12.492	15.739	16.955	21.362	26.914	33.910	46.022
0.100	3.620	4.561	7.800	9.827	12.382	13.338	16.805	21.173	26.676	36.205
0.150	3.133	3.948	6.751	8.505	10.716	11.544	14.544	18.324	23.087	31.334
0.200	2.819	3.552	6.074	7.653	9.642	10.386	13.086	16.487	20.772	28.192
0.250	2.590	3.264	5.581	7.031	8.859	9.543	12.023	15.148	19.086	25.903
0.300	2.411	3.038	5.195	6.545	8.246	8.383	11.192	14.101	17.766	24.112
0.350	2.264	2.853	4.878	6.146	7.743	8.341	10.509	13.241	16.682	22.641
0.400	2.139	2.695	4.609	5.806	7.316	7.881	9.929	12.510	15.761	21.391
0.450	2.030	2.557	4.373	5.510	6.942	7.478	9.422	11.871	14.956	[illegible]
0.500	[illegible]	2.434	4.165	5.245	[illegible]	7.118	8.968	11.299	14.236	19.322

Table 7: Scatter Factors for α = 3.0

ALFA= 4.00

BETA= 5094.8

GAMA	N= 5	N= 10	N= 50	N= 100	N= 200	N= 250	N= 500	N=1000	N=2000	N=5000
0.010	4.723	5.616	8.398	9.987	11.877	12.559	14.935	17.760	21.121	26.558
0.050	3.142	3.737	5.588	6.645	7.902	8.355	9.936	11.816	14.052	17.670
0.100	2.625	3.121	4.667	5.550	6.601	6.979	8.300	9.870	11.738	14.760
0.150	2.355	2.801	4.188	4.981	5.923	6.263	7.448	8.857	10.533	13.244
0.200	2.176	2.587	3.869	4.601	5.472	5.785	6.880	8.182	9.730	12.235
0.250	2.042	2.428	3.631	4.318	5.135	5.429	6.457	7.678	9.131	11.482
0.300	1.935	2.301	3.441	4.092	4.866	5.145	6.119	7.277	8.653	10.881
0.350	1.846	2.195	3.282	3.903	4.642	4.908	5.837	6.941	8.255	10.380
0.400	1.769	2.103	3.145	3.741	4.448	4.703	5.593	6.652	7.910	9.947
0.450	1.701	2.022	3.024	3.596	4.277	4.522	5.378	6.395	7.605	9.563
0.500	1.639	1.949	2.914	3.466	4.121	4.358	5.182	6.163	7.329	9.216

Table 8: Scatter Factors for $\alpha = 4.0$

ALFA= 5.00

BETA= 5129.0

GAMA	N= 5	N= 10	N= 50	N= 100	N= 200	N= 250	N= 500	N=1000	N=2000	N=5000
0.010	3.462	3.977	5.487	6.303	7.241	7.571	8.697	9.990	11.475	13.783
0.050	2.499	2.871	3.961	4.550	5.226	5.465	6.277	7.211	8.283	9.949
0.100	2.164	2.486	3.430	3.940	4.526	4.732	5.436	6.244	7.172	8.615
0.150	1.984	2.279	3.145	3.613	4.150	4.339	4.984	5.726	6.577	7.900
0.200	1.862	2.139	2.952	3.391	3.895	4.073	4.678	5.374	6.173	7.414
0.250	1.770	2.033	2.806	3.223	3.702	3.871	4.446	5.108	5.867	7.047
0.300	1.696	1.948	2.687	3.087	3.546	3.708	4.259	4.893	5.620	6.751
0.350	1.633	1.876	2.588	2.973	3.415	3.571	4.101	4.711	5.412	6.500
0.400	1.578	1.813	2.501	2.873	3.300	3.451	3.964	4.554	5.231	6.283
0.450	1.529	1.757	2.424	2.784	3.198	3.344	3.841	4.412	5.069	6.088
0.500	1.485	1.705	2.353	2.703	3.105	3.247	3.729	4.284	4.921	5.911

Table 9: Scatter Factors for α = 5.0

ALFA= 6.00

BETA= 5159.2

GAMA	N= 5	N= 10	N= 50	N= 100	N= 200	N= 250	N= 500	N=1000	N=2000	N=5000
0.010	2.815	3.160	4.132	4.638	5.206	5.403	6.065	6.807	7.641	8.902
0.050	2.145	2.408	3.149	3.534	3.967	4.118	4.622	5.188	5.823	6.734
0.100	1.903	2.136	2.793	3.135	3.519	3.652	4.099	4.601	5.165	6.017
0.150	1.770	1.987	2.598	2.916	3.274	3.398	3.814	4.281	4.805	5.598
0.200	1.679	1.865	2.465	2.766	3.105	3.223	3.617	4.060	4.558	5.310
0.250	1.609	1.807	2.362	2.652	2.976	3.089	3.467	3.892	4.369	5.090
0.300	1.553	1.743	2.279	2.558	2.872	2.980	3.345	3.755	4.215	4.910
0.350	1.505	1.689	2.209	2.479	2.783	2.888	3.242	3.639	4.084	4.758
0.400	1.463	1.642	2.147	2.410	2.705	2.807	3.151	3.537	3.970	4.625
0.450	1.425	1.599	2.091	2.347	2.635	2.735	3.069	3.445	3.867	[illegible]
0.500	1.390	[illegible]	[illegible]	2.290	2.571	2.606	2.945	3.361	3.773	4.396

Table 10: Scatter Factors for α = 6.0

ALFA= 8.00

BETA= 5208.9

GAMA	N= 5	N= 10	N= 50	N= 100	N= 200	N= 250	N= 500	N=1000	N=2000	N=5000
0.010	2.173	2.370	2.898	3.160	3.446	3.544	3.865	4.214	4.596	5.153
0.050	1.773	1.933	2.364	2.578	2.811	2.891	3.152	3.437	3.749	4.204
0.100	1.620	1.767	2.160	2.356	2.569	2.642	2.881	3.142	3.426	3.842
0.150	1.535	1.674	2.046	2.232	2.434	2.503	2.729	2.976	3.245	3.639
0.200	1.475	1.609	1.967	2.145	2.339	2.405	2.623	2.860	3.119	3.498
0.250	1.429	1.558	1.905	2.078	2.266	2.330	2.541	2.771	3.022	3.388
0.300	1.391	1.517	1.855	2.023	2.206	2.268	2.474	2.698	2.942	3.299
0.350	1.359	1.482	1.812	1.976	2.154	2.215	2.416	2.635	2.373	3.222
0.400	1.330	1.450	1.774	1.934	2.109	2.169	2.365	2.579	2.813	3.154
0.450	1.304	1.422	1.739	1.896	2.068	2.127	2.319	2.529	2.758	3.092
0.500	1.280	1.396	1.707	1.862	2.030	2.088	2.277	2.483	2.707	3.036

Table 11: Scatter Factors for $\alpha = 8.0$

ALFA= 10.00

BETA= 5247.1

GAMA	N= 5	N= 10	N= 50	N= 100	N= 200	N= 250	N= 500	N=1000	N=2000	N=5000
0.010	1.861	1.994	2.342	2.511	2.691	2.752	2.949	3.161	3.388	3.713
0.050	1.581	1.694	1.990	2.133	2.286	2.338	2.505	2.685	2.878	3.154
0.100	1.471	1.577	1.852	1.965	2.127	2.175	2.331	2.499	2.675	2.935
0.150	1.409	1.510	1.773	1.901	2.037	2.083	2.233	2.393	2.565	2.811
0.200	1.365	1.463	1.718	1.841	1.974	2.018	2.163	2.318	2.485	2.723
0.250	1.330	1.426	1.675	1.795	1.924	1.967	2.109	2.260	2.422	2.655
0.300	1.302	1.396	1.639	1.757	1.883	1.926	2.064	2.212	2.371	2.598
0.350	1.276	1.370	1.609	1.724	1.848	1.890	2.025	2.171	2.326	2.550
0.400	1.256	1.346	1.582	1.695	1.817	1.858	1.991	2.134	2.287	2.507
0.450	1.237	1.325	1.557	1.669	1.788	1.829	1.960	2.101	2.251	2.467
0.500	1.218	1.306	1.534	1.644	1.762	1.802	1.931	2.070	2.218	2.431

Table 12: Scatter Factors for α = 10.0

ALFA= 15.00

BETA= 5309.5

GAMA	N= 5	N= 10	N= 50	N= 100	N= 200	N= 250	N= 500	N=1000	N=2000	N=5000
0.010	1.513	1.564	1.764	1.847	1.935	1.964	2.056	2.154	2.256	2.398
0.050	1.357	1.421	1.582	1.657	1.735	1.761	1.845	1.932	2.023	2.151
0.100	1.293	1.355	1.508	1.579	1.654	1.679	1.758	1.841	1.929	2.050
0.150	1.257	1.316	1.465	1.534	1.607	1.631	1.708	1.789	1.874	1.992
0.200	1.230	1.289	1.434	1.502	1.573	1.597	1.672	1.752	1.834	1.950
0.250	1.210	1.267	1.410	1.477	1.547	1.570	1.644	1.722	1.804	1.917
0.300	1.192	1.249	1.390	1.456	1.525	1.548	1.621	1.698	1.778	1.890
0.350	1.178	1.233	1.373	1.438	1.506	1.528	1.601	1.676	1.755	1.866
0.400	1.164	1.219	1.357	1.422	1.489	1.511	1.583	1.657	1.736	1.845
0.450	1.152	1.207	1.343	1.407	1.473	1.495	1.566	1.640	1.718	[illegible]
0.500	1.141	1.195	1.330	1.393	1.459	1.481	1.551	1.624	1.701	1.808

Table 13: Scatter Factors α = 15.0

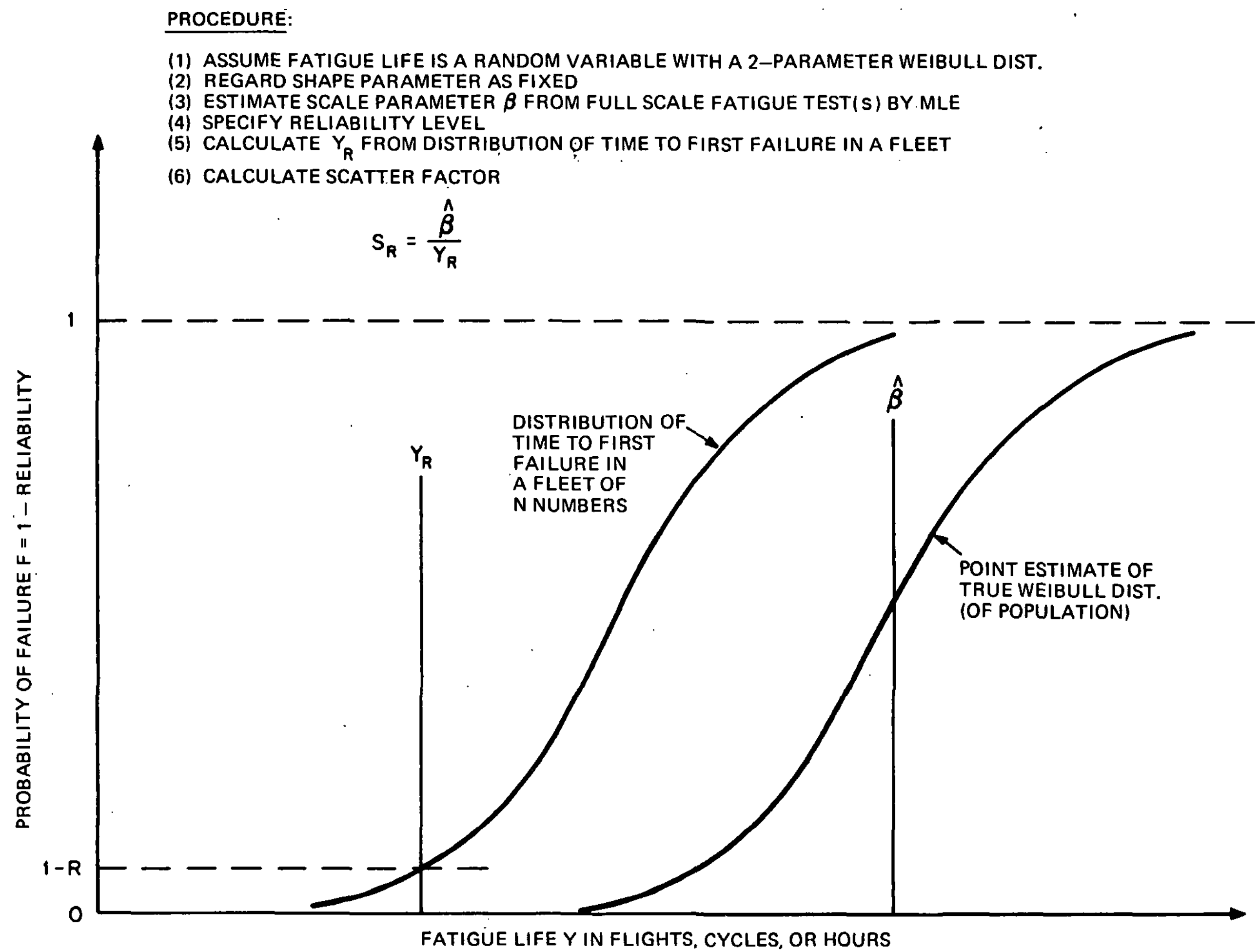

FIGURE 1 SCHEMATIC REPRESENTATION OF RELIABILITY PLAN

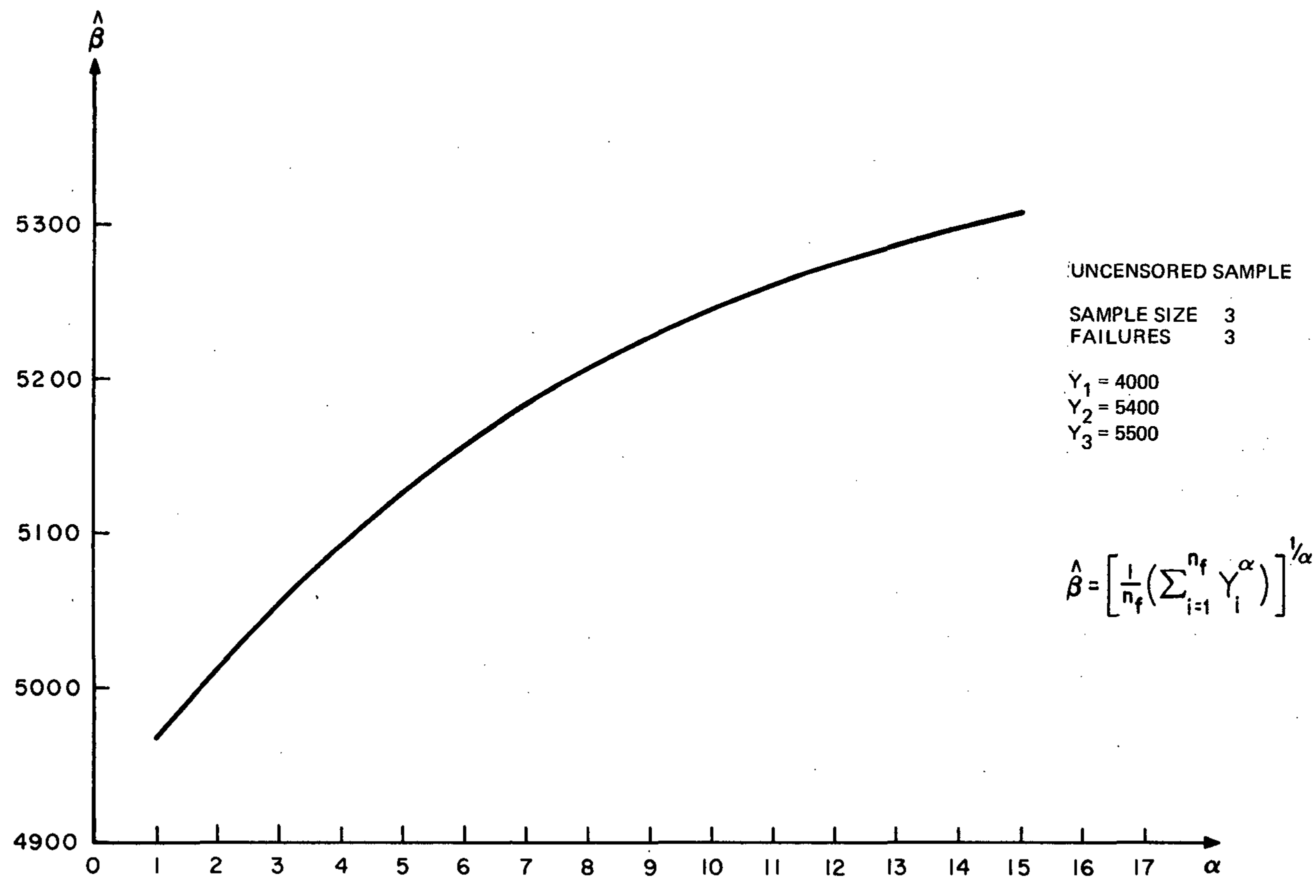

FIGURE 2 CHARACTERISTIC LIFE MLE $\hat{\beta}$ AS FUNCTION OF α

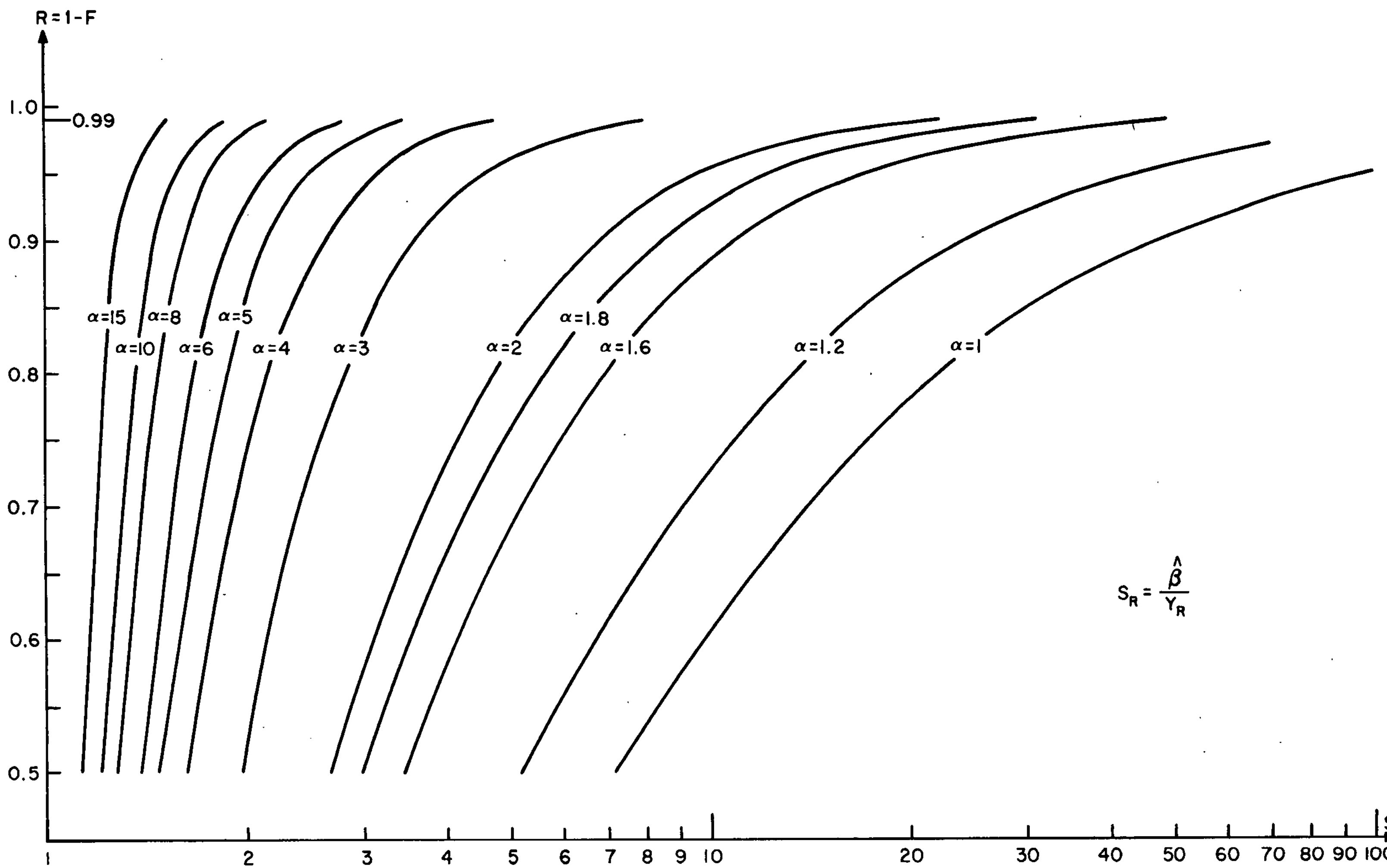

FIGURE 3 SCATTER FACTORS FOR FLEETSIZE N=5

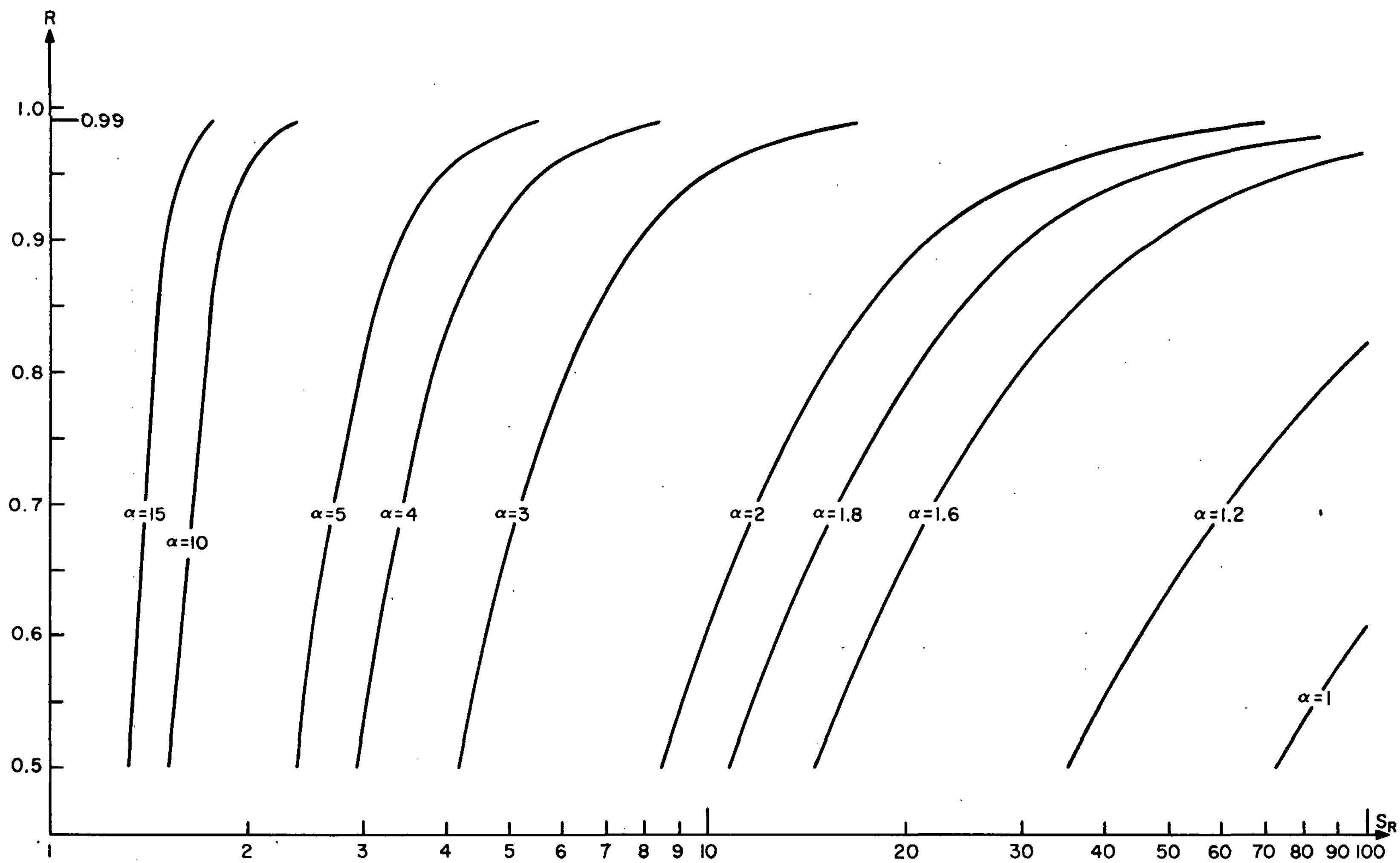

FIGURE 4 SCATTER FACTORS FOR FLEETSIZE N=50

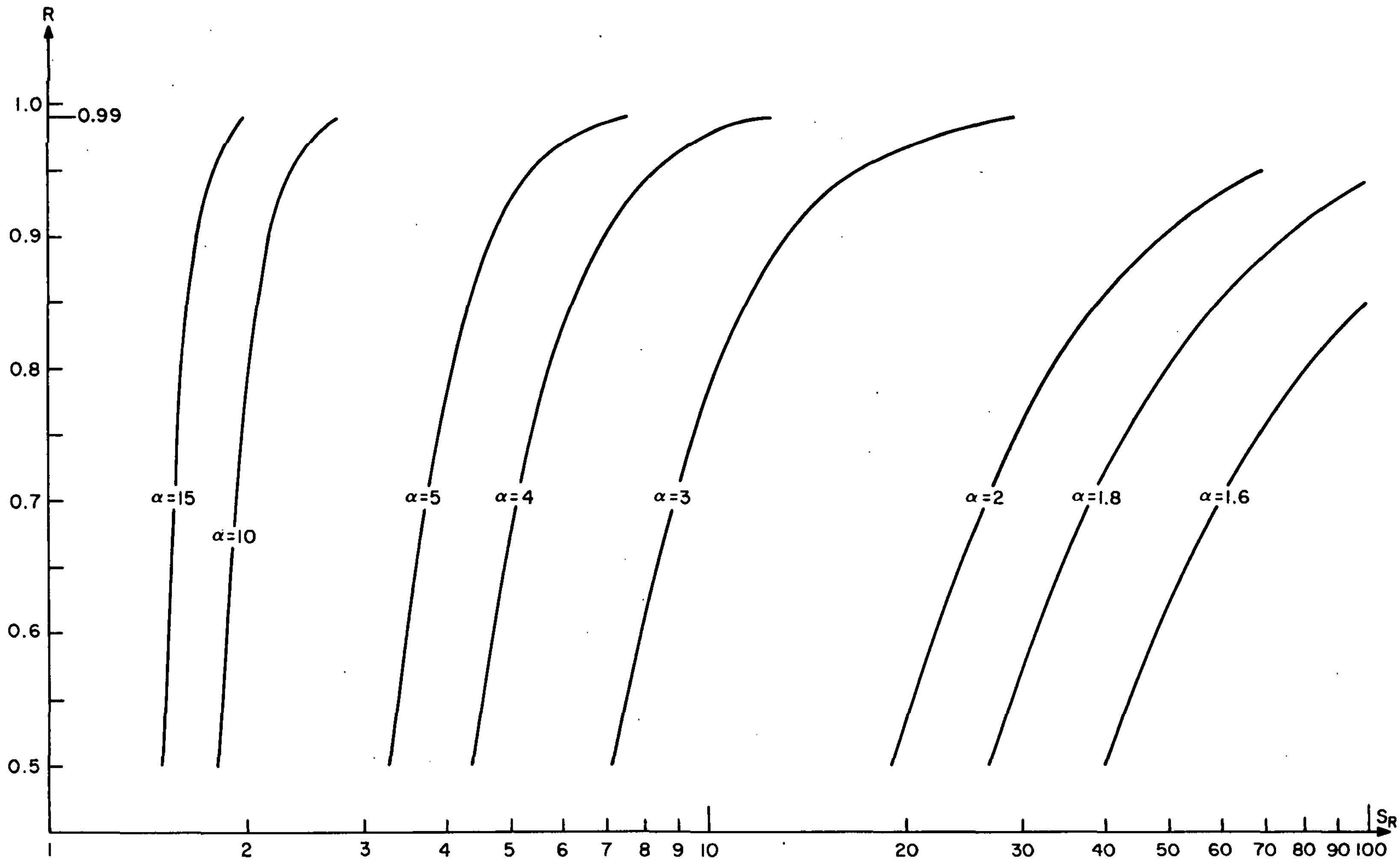

FIGURE 5 SCATTER FACTORS FOR FLEETSIZE N=250

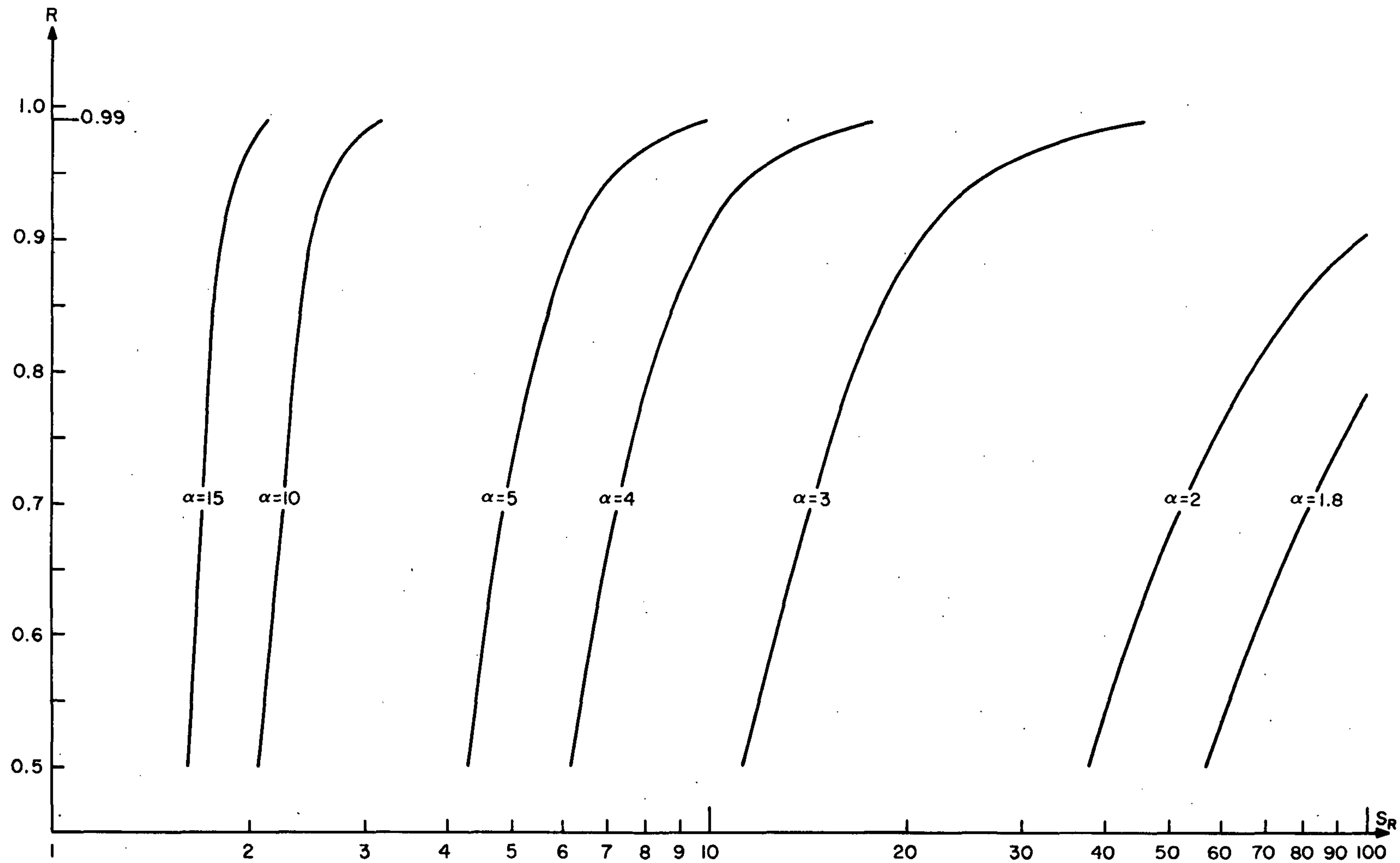

FIGURE 6 SCATTER FACTORS FOR FLEETSIZE N=1000

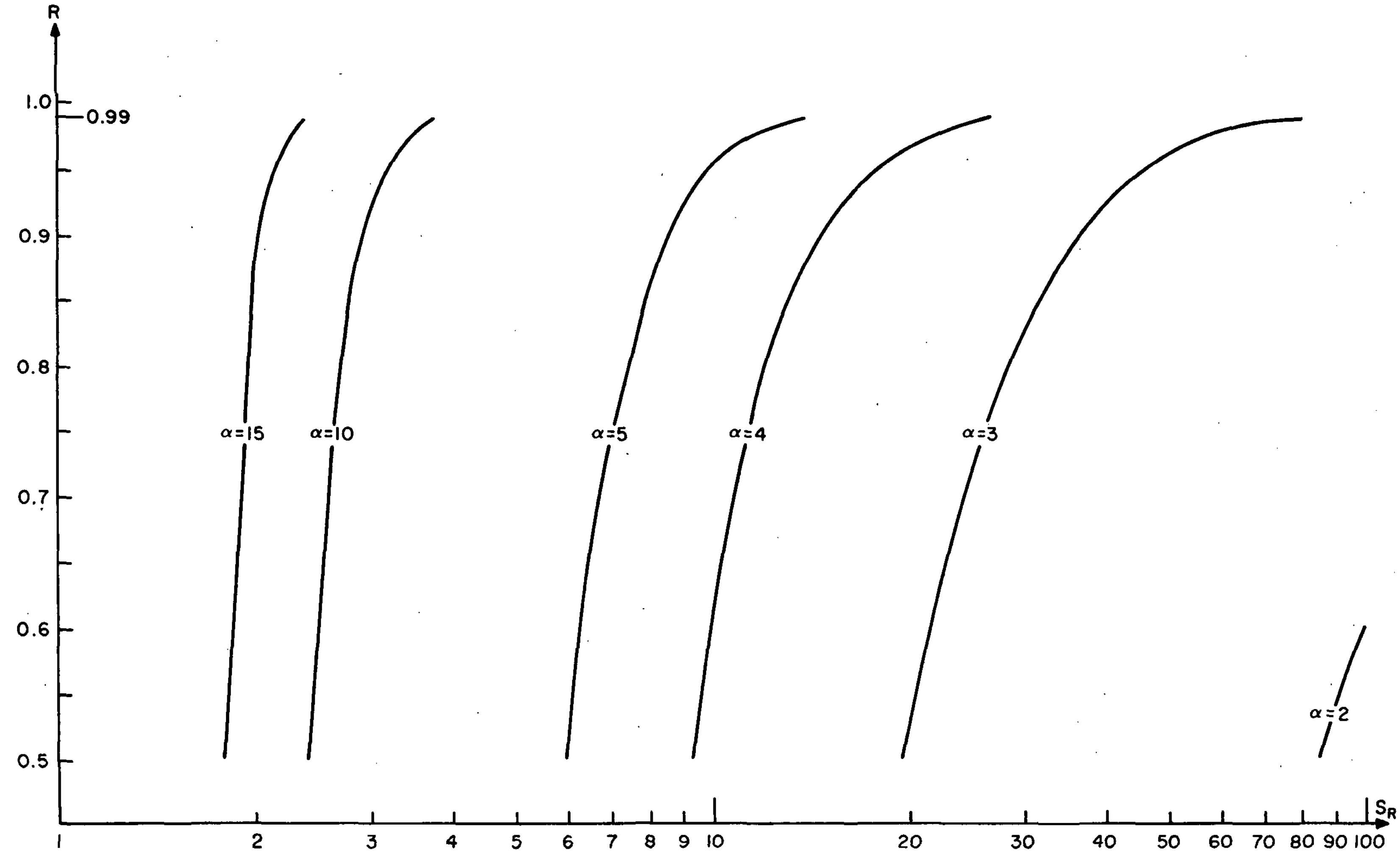

FIGURE 7 SCATTER FACTORS FOR FLEETSIZE N=5000

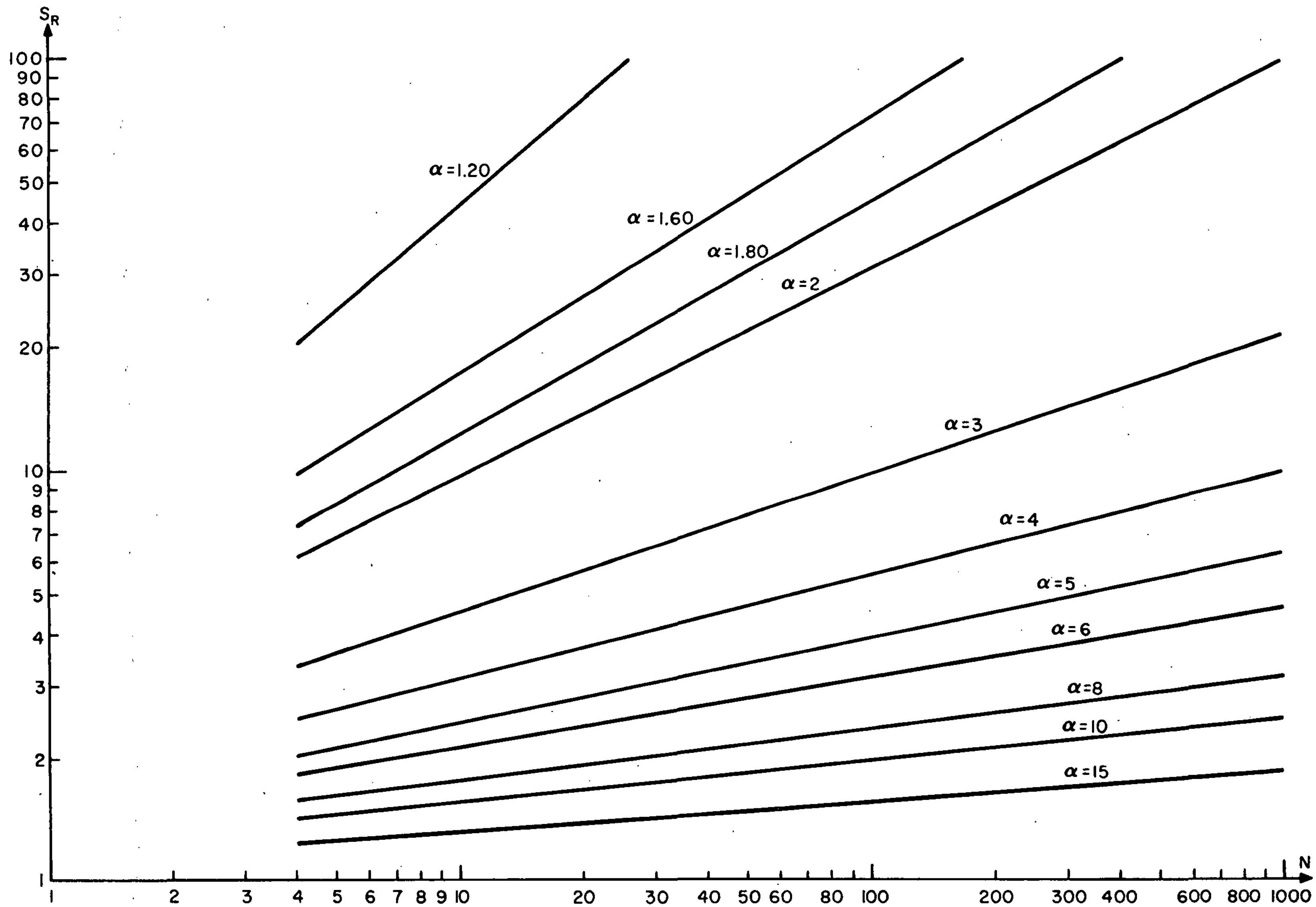

FIGURE 8 SCATTER FACTORS FOR RELIABILITY R=0.90

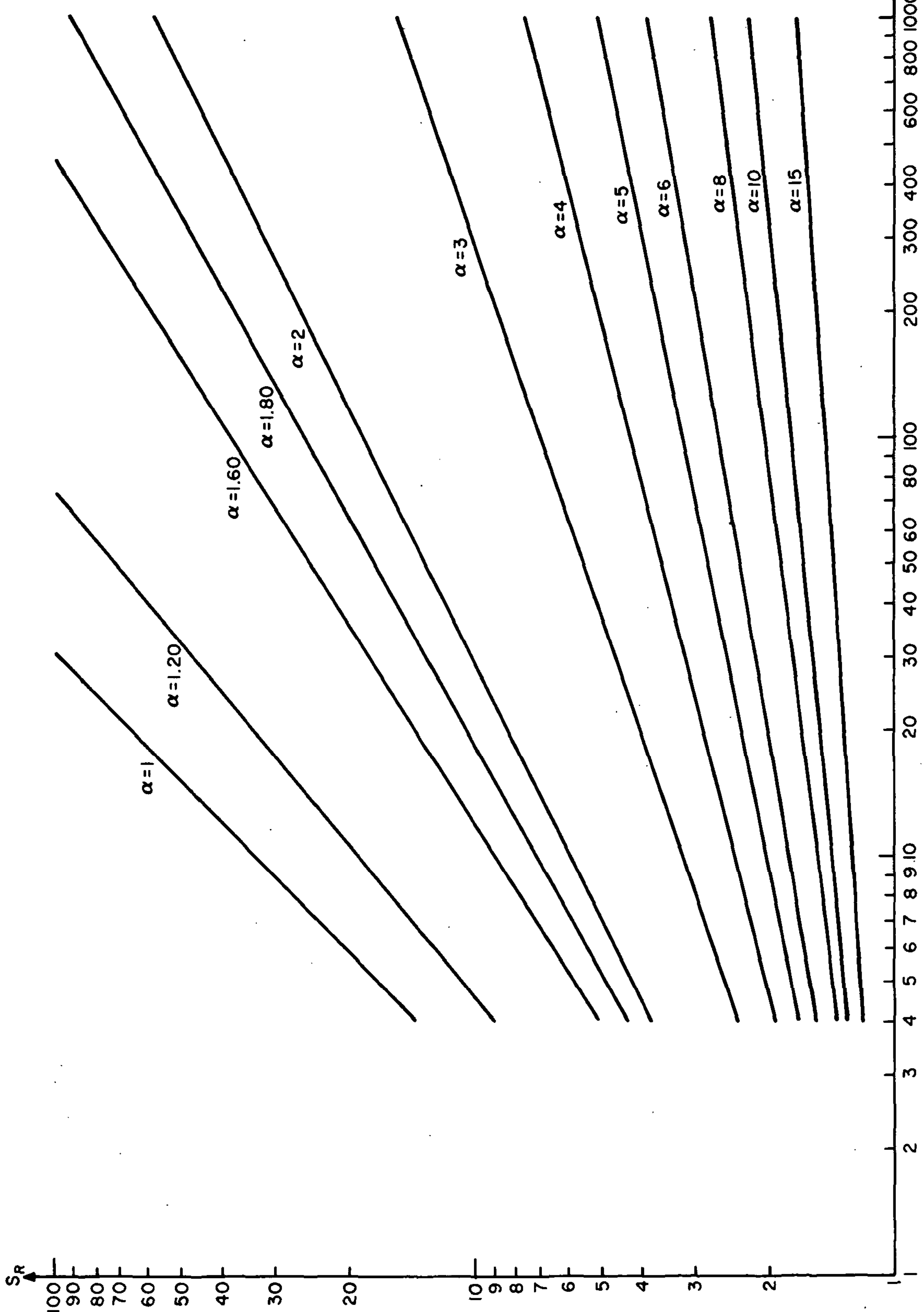

FIGURE 9 SCATTER FACTORS FOR RELIABILITY R=0.75

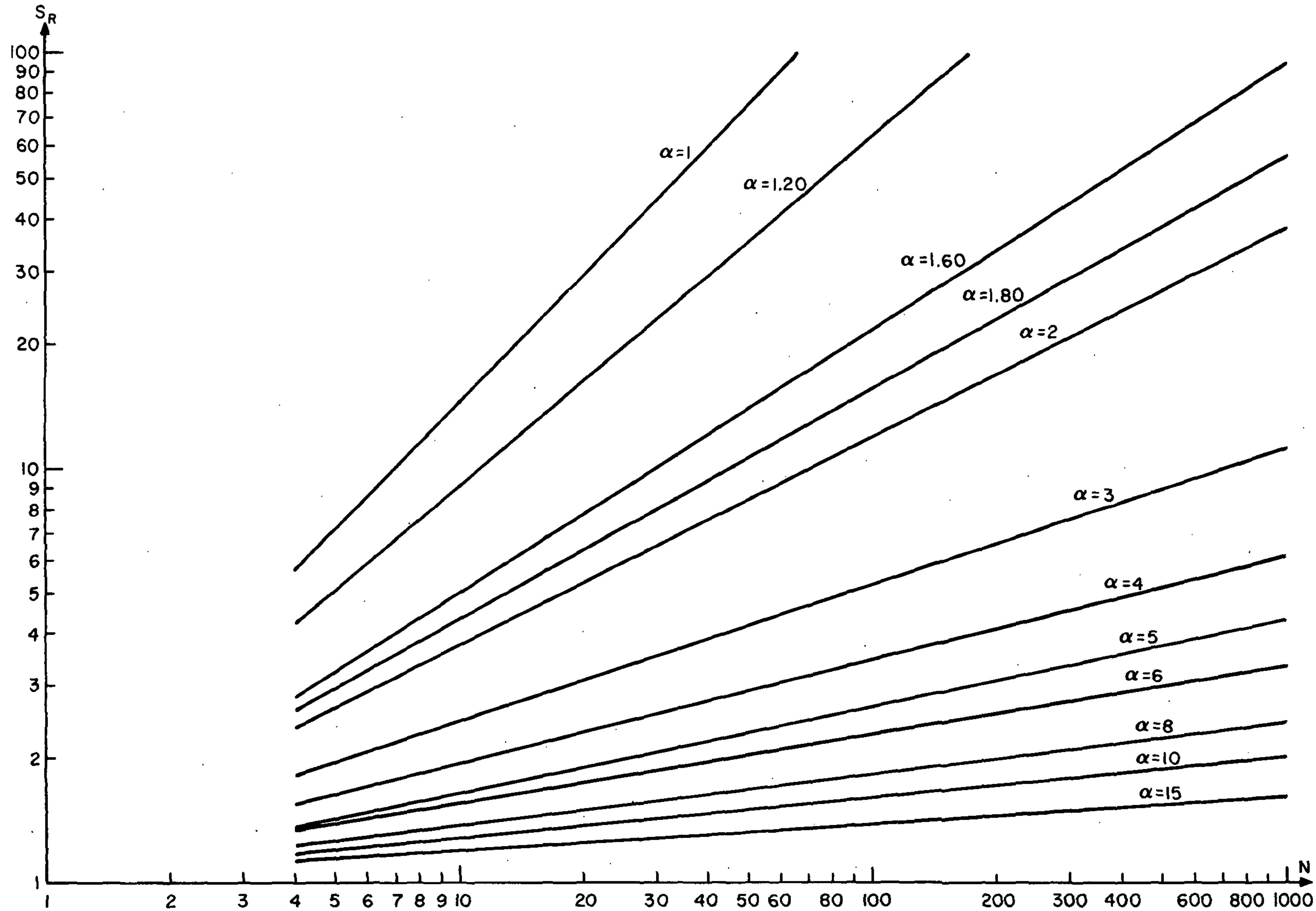

FIGURE 10 SCATTER FACTORS FOR RELIABILITY R=0.50

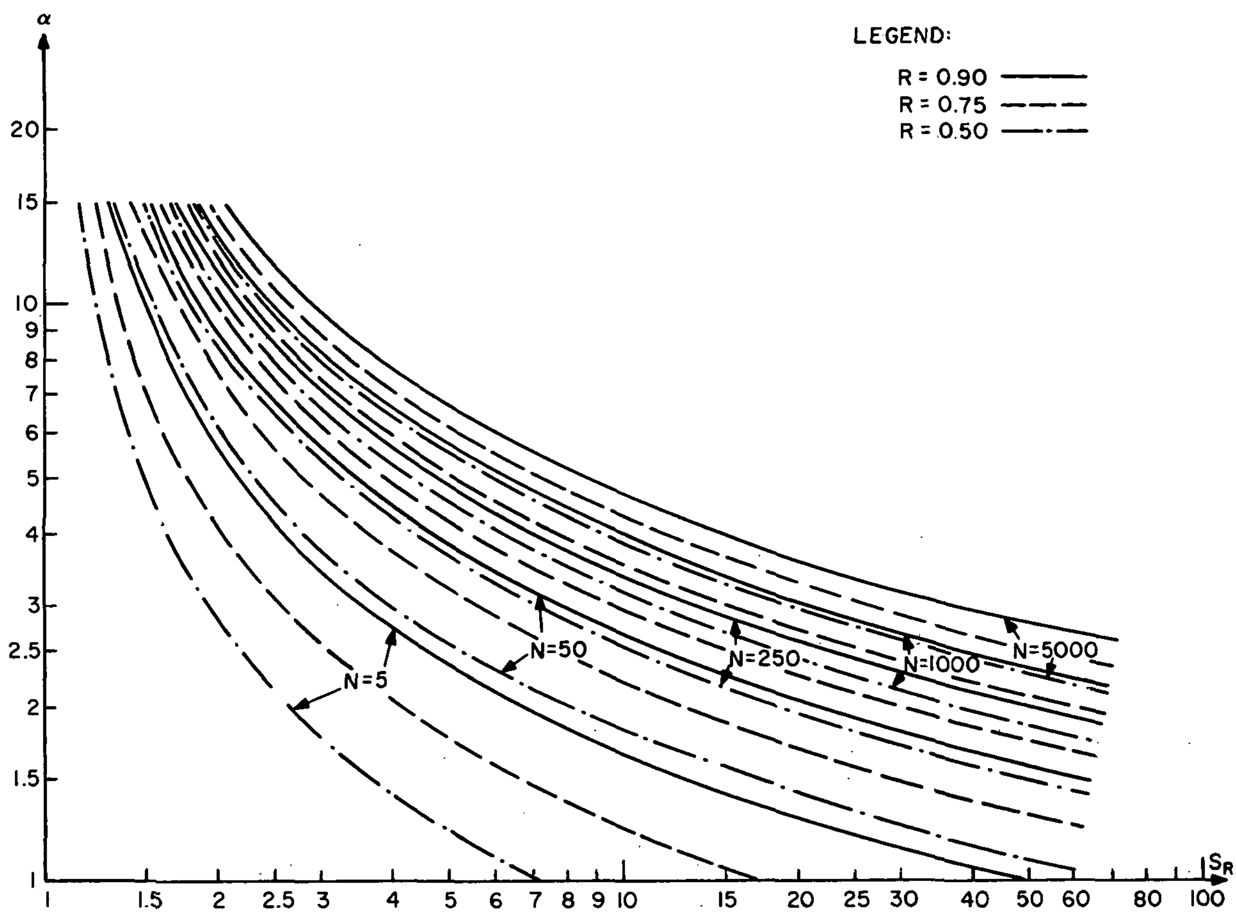

FIGURE 11 SHAPE PARAMETER VERSUS SCATTER FACTOR

```
$JOB (7934,F),'SCHUELLER',PAGES=30
C *** SCATTER FACTOR AND THE RELIABILITY ANALYSIS OF AIRCRAFT STRUCTURES
C     THIS PROGRAM CALCULATES SCATTER FACTORS FOR VARIOUS PARAMETERS(N,ALFA,
C     GAMA)
C
C *** VARIABLES
C
C     SF = SCATTER FACTOR
C     GAMA = CONFIDENCE INTERVAL FOR FIRST FAILURE DISTRIBUTION
C     NG = NUMBER OF GAMMAS USED IN PROGRAM
C     FN = FLEETSIZE
C     NFN = NUMBER OF FLEETSIZES
C     NS = SAMPLE SIZE
C     NF = NUMBER OF FAILURES
C     ALFA = WEIBULL SHAPE PARAMETER
C      Y = FATIGUE TEST SAMPLE
C      BETA = MLE OF WEIBULL SCALE PARAMETER(POPULATION) UNBIASED
C      BETA1 = MLE OF WEIBULL SCALE PARAMETER (FIRST FAILURE) UNBIASED
C      NA = NUMBER OF ALFAS
C      ZR1 = ESTIMATED LIFE FOR SPECIFIC CONFIDENCE INTERVAL(FIRST FAILURE)
C     YF = FATIGUE FAILURE SAMPLE
C     YNF = NF FAILURE TIME
C     X1 = TIME TO FIRST FAILURE
C
C *** DECLARATIONS
C
      INTEGER FN
      DIMENSION A(100),B(100)
       DIMENSION GAMA(20),ZR1(200),BETA1(200),BETA(200)
      DIMENSION FN(50),YF(20),Y(20),ALFA(50)
      DIMENSION SF(200),W(20)
C
C*** INPUT - INFORMATION
C
      READ (5,500) NS,NF,NA,NG,NFN
  500 FORMAT (5I10)
      WRITE (6,501)
  501 FORMAT (1H1,/////,65X,'I N P U T ',///)
      WRITE (6,502) NS,NF,NA,NG,NFN
  502 FORMAT (5X,'SAMPLESIZE=',I3,3X,'NUMBER FAILED=',I3,3X,'ALFAS=',
     XI3,3X,'GAMAS=',I3,3X,'NUMBER OF FLEETS=',I3,///)
      READ (5,504) (Y(I),I = 1,NS)
  503 FORMAT (10X,'S A M P L E S',/)
      WRITE (6,503)
      WRITE (6,504) (Y(I),I=1,NS)
  504 FORMAT (10F8.1)
      READ (5,504) (YF(I),I=1,NF)
  505 FORMAT (//,10X,' F A I L U R E S',/)
      WRITE (6,505)
      WRITE (6,504) (YF(I),I = 1,NF)
      READ (5,700)YNF
 700  FORMAT (F10.1)
      READ (5,506) (ALFA(I),I = 1,NA)
      WRITE (6,520)
  520 FORMAT (//,10X,'A L F A S',/)
  506 FORMAT (8F10.2)
      WRITE (6,506) (ALFA(I),I = 1,NA)
       WRITE(6,507)
  507 FORMAT (//,10X,'F L E E T S I  Z E S',/)
      READ (5,508) (FN(I), I=1,NFN)
```

```
  508 FORMAT (16I5)
      WRITE (6,508) (FN(I), I=1,NFN)
         WRITE (6,509)
  509 FORMAT (///,6X,'GAMA',7X,'W',//)
      DO 1 I = 1,NG
      READ (5,510) GAMA(I),W(I)
  510 FORMAT (2F10.3)
      WRITE (6,511) I,GAMA(I),W(I)
  511 FORMAT (1X,I2,F7.3,F10.3)
    1 CONTINUE
C
C *** ESTIMATE WEIBULL SCALE PARAMETER
C
C    *(1) UNZENSORED SAMPLE*
C
      DO 2 J = 1,NA
      WRITE (6,518) ALFA(J)
  518 FORMAT (1H1,///,65X,'ALFA=',F6.2,/)
      IF (NS.NE.NF) GO TO 4
      SUMY = 0.0
      DO 3 L = 1,NS
      SUMY     = SUMY     + (Y(L)**ALFA(J))
    3 CONTINUE
      BETA(J) = (1./NF*SUMY    )**(1./ALFA(J))
      WRITE (6,512) BETA(J)
  512 FORMAT (     //,10X,'BETA=',F10.1,//)
      GO TO 5
    4 CONTINUE
C
C *** (2) ZENSORED SAMPLE
C
      SUMY = 0.0
      DO 6 I=1,NS
      SUMY     = SUMY     + (Y(L)**ALFA(J))
    6 CONTINUE
      BETA(J) =(1./NF*(SUMY + ((NS-NF) *(YNF**ALFA(J)))))**(1./ALFA(J))
      WRITE (6,519) BETA(J)
  519 FORMAT (     //,10X,'BETA=',F10.1,//)
    5 CONTINUE
C
C *** CALCULATE X1(GAMA)
C
      WRITE (6,515) (FN(I), I=1,NFN)
  515 FORMAT (//,3X,'GAMA',7X,'N=',I2,5X,'N=',I3,5X,'N=',I3,4X,'N=',I4,
     X4X,'N=',I4,4X,'N=',I4,4X,'N=',I4,4X,'N=',I4,4X,'N=',I4,4X,'N=',
     XI4,///)
      DO 8 K = 1,NG
      DO 7 I = 1,NFN
      BETA1(I) = BETA(J) * 1./(FN(I)**(1./ALFA(J)))
      A(K) = (1.0 - GAMA(K))
      B(J) = 1.0 / ALFA(J)
      ZR1(I) = BETA1(I)*((ABS(ALOG(A(K))))**(B(J)))
      SF(I) = BETA(J)/ZR1(I)
    7 CONTINUE
      WRITE (6,513) GAMA(K),(SF(I),I=1,NFN)
  513 FORMAT (F8.3,10(F10.3))
    8 CONTINUE
    2 CONTINUE
      RETURN
      END
```


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